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Expulsion of Magnetic Flux Lines from the Growing Superconducting Core of a Magnetised Quark Star

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Abstract The expulsion of magnetic flux lines from a growing superconducting core of a quark star has been investigated. The idea of impurity diffusion in molten alloys and an identical mechanism of baryon number transport from hot quark-gluon-plasma phase to hadronic phase during quark-hadron phase transition in the early universe, micro-second after big bang has been used. The possibility of Mullins-Sekerka normal-superconducting interface instability has also been studied.

Keywords
Quark star, superconductivity, flux expulsion, Mullins-Sekerka instability

1 Introduction

If the matter density at the core of a neutron star exceeds a few times normal nuclear density (e.g. $> 3n_0$, where $n_0 = 0.17\text{fm}^{-3}$, the normal nuclear density), a deconfining phase transition to quark matter may occur. As a consequence a normal neutron star will be converted to a hybrid star with an infinite cluster of quark matter core and a crust of neutron matter. If the speculation of Witten [1] that a flavour symmetric quark matter may be the absolute ground state at zero temperature and pressure is assumed to be correct, then it may be quite possible that the whole neutron star / hybrid star will be converted to a star of quark matter, known as quark star or strange star.

From the observed features in the spectra of pulsating accreting neutron stars in binary system, the strength of surface magnetic field of a normal neutron star is found to be $\sim 10^{12}$G [2,3,4,5]. At the core region of a neutron star it probably reaches a few orders of magnitude more than the surface value. In some publications several years ago we have shown that if the intensity of magnetic field at the core region of a compact neutron star exceeds some critical value which is the typical strength of magnetic field at which the cyclotron lines begin to appear or equivalently at which the cyclotron quantum is of the order of or greater than the rest mass of the particle considered or the de Broglie wave length is of the order of or greater than the Larmor radius of the particle, then there can not be nucleation of any quark matter bubble in the metastable neutron matter [6,7]. The surface as well as the curvature energies of the new phase diverge in this case. As a consequence the new quark matter phase can not be thermodynamically favourable over the metastable neutron matter phase. Therefore to achieve a first order deconfining transition initiated by the nucleation of quark droplets at the core of neutron star, it has to be assumed that the strength of magnetic field throughout the star is much less than the corresponding critical value. In the case of electron of mass 0.5MeV, this critical field is $\sim 4.4 \times 10^{13}$G, for light quarks of current mass 5MeV, it is $\sim 10^{15}$G, whereas for s-quark of current mass 150MeV, it is $\sim 10^{20}$G.

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Now for a many body fermion system, the microscopic theory of superconductivity suggests that if the interaction favours formation of pairs at low temperature [8], the system may undergo a phase transition to a super-fluid state. This is expected to occur in the dense neutron matter present in neutron star [9,10,11]. On the other hand, if the particles carry charges, the paired state will be superconducting (neutron matter becomes super-fluid, whereas the small percentage of protons undergoes a transition to type-II superconducting phase). In the case of a many body system of electrons the well known BCS theory is generally used to study the superconducting properties due to electron pairing [8]. One electron of momentum $\mathbf{k}$ and spin $\mathbf{s}$ combines with another one of momentum $-\mathbf{k}$ and spin $-\mathbf{s}$ and form a Cooper pair. The coupling is mediated by the electron-phonon interaction. In the case of quark matter, the basic quark-quark interaction is attractive at large distances and consequently the BCS pairing mechanism is also applicable here. For a highly degenerate system, which is true in quark star, the pairing takes place near the Fermi surface. The other condition that must be satisfied to form Cooper pairs is that the temperature (T) of the system should be much less than the superconducting energy gap ($\varDelta$), which is a function of the interaction strength and the density of the system. This is the most important criterion for the occurrence of superconducting transition. In the case of quark stars, only quarks can form Cooper pairs. Whereas the electrons, whose density is extremely low compared to quark matter, may be treated as highly degenerate relativistic plasma and are unlikely to form Cooper pairs (since the quark matter is in β-equilibrium, a small fraction of electrons should be present in the system). The kinetic energy of the electronic part dominates over its attractive potential energy (which is electromagnetic in nature), and as a result the corresponding superconducting transition temperature should be extremely low and in reality may not be achieved in a quark star. The relativistic version of the theory of super-fluidity and superconductivity for a system of many fermions was developed long ago by Bailin and Love [12]. In that paper, an overview of mathematical formalism for relativistic version of BCS theory is given and discussed with elaborate mathematical derivation to obtain the critical field(s) for both type-I and type-II superconductors. They have also given a rigorous mathematical derivation to get the critical temperature above which the superconducting property is completely destroyed and also obtained the expressions for correlation lengths.

In the past few years a lot of works have also been done on a new concept, the possibility of colour superconductivity in quark stars with light flavor pairing, known as $2SC$, with unpaired s-quarks. A lot of works have also been done on the possibility of an entirely new kind of phase, known as the *Colour-Flavor Locked* (CFL) phase, which is colour neutral and charge neutral system and is also expected to be energetically more favourable than strange quark matter [13,14,15,16,17,18,19,20]. However, the density at which this new phase appears is several times normal nuclear density.

In the present article we have assumed a type-I superconducting phase transition in quark matter at the core region of a quark star and investigated the mechanism by which the magnetic flux lines are expelled from the superconducting zone.

In a very recent work by Konenkov and Geppert have investigated the expulsion of magnetic flux lines from superconducting core region of neutron stars [21]. They have considered a type-II superconducting transition at the core region and studied the movement of quantized fluxoids. They have also given a mechanism by which the flux lines expelled from the core into the crustal region undergo ohmic decay.

Now the quarks with identical Fermi energy can only combine to form Cooper pairs at the Fermi surface. Since for u and d quarks, the current masses are equal and also their chemical potentials are almost identical, whereas s quark is much more heavier than u and d quarks and also its chemical potential is different, we may therefore have only uu, dd, ud and ss Cooper pairs in the system. For iso-spin 1/2 flavours, the contribution may come either from iso-scalar or iso-vector channels. It was shown in ref.[12] that the pairing of $u-u$, $d-d$ or $u-d$ system will be favoured by iso-scalar combination rather than iso-vector channel. On the other hand the $s-s$ combination is a triplet state with $J^P = 1^+$. It has been shown in ref.[12] that if a normal quark matter system undergoes a superconducting phase transition, the newly produced quark matter phase will be a type-I superconductor. They have also shown in that paper that the critical magnetic field to destroy such pairing is $\sim 10^{16}$G for $n \sim 2-3n_0$, which is indeed much larger than the typical pulsar magnetic field The corresponding critical temperature is $\sim 10^9 - 10^{10}$K, which is again high enough for quark stars / core of hybrid stars, which are expected to be extremely cold objects. (in this connection we would like to mention that in the present article we are not going to study the superconducting properties of the matter inside the strange stellar objects, called the magnetars, with the surface magnetic field strength

$\geq 10^{15}$G). Therefore, we may expect that the magnetic field strengths at the core region of a quark star are much less than the corresponding critical value for the destruction of superconducting property and the temperature is also low enough. Then during such a type-I superconducting phase transition, the magnetic flux lines from the superconducting quark sector of the quark star will be pushed out towards the normal crustal region. Now for a small type-I superconducting laboratory sample placed in an external magnetic field less than the corresponding critical value, the expulsion of magnetic field takes place instantaneously. Whereas in the quark star scenario, the picture may be completely different. It may take several thousands of years for the magnetic flux lines to get expelled from the superconducting core region. Which further means, that the growth of superconducting phase in quark stars will not be instantaneous. Therefore in a quark star / hybrid star, with type-I superconducting quark matter at the core, the magnetic flux lines will be completely expelled by Meissner effect not instantaneously, it takes several thousand years of time. A simple estimate shows that the expulsion time due to ohmic diffusion is $\sim 10^4$yrs [22]. It was shown by Chau using Ginzberg-Landau formalism that the time for expulsion of magnetic lines of force accompanied by the enhancement of magnetic field non-uniformities at the crustal region gets prolonged to 10^7yrs [23]. Alford et. al. investigated the expulsion of magnetic lines of force from the colour superconducting region by considering the pairing of like and unlike quarks and obtained the expulsion time much larger than the age of the Universe [24].

The aim of the present article is to investigate the expulsion of magnetic flux lines from the growing superconducting core of a quark star. We have used the idea of impurity diffusion in molten alloys or the transport of baryon numbers from hot quark matter soup to hadronic matter during quark-hadron phase transition in the early universe, micro-second after big bang (the first mechanism is used by the material scientists and metallurgists [25], whereas the later one is used by cosmologists working in the field of big bang nucleo-synthesis [26,27]), We have also studied the possibility of Mullins-Sekerka normal-superconducting interface instability [28,29] in quark matter. This is generally observed in the case of solidification of pure molten metals at the solid-liquid interface, if there is a temperature gradient. The interface will always be stable if the temperature gradient is positive and otherwise it will be unstable. In alloys, the criteria for stable / unstable behaviour is more complicated. It is seen that, during the solidification of an alloy, there is a substantial change in the concentration ahead of the interface. Here solute diffusion as well as the heat flow effects must be considered simultaneously. The particular problem we are going to investigate here is analogous to solute diffusion during solidification of an alloy.

2 Formalism

It has been assumed that the growth of superconducting quark bubble started from the centre of the star and the nomenclature *controlled growth* for such phenomenon has been used. If the magnetic field strength and the temperature of the star are a few orders of magnitude less than their critical values, the normal quark matter phase is thermodynamically unstable relative to the corresponding superconducting one. Then due to fluctuation, a droplet of superconducting quark matter bubble may be nucleated in metastable normal quark matter medium. If the size of this superconducting bubble is greater than the corresponding critical value, it will act as the nucleating centre for the growth of superconducting quark core. The critical radius can be obtained by minimising the free energy. Then following the work of Mullins and Sekerka, we have [28,29]

$$r_c = \frac{16\pi\alpha}{B_m^{(c)2}\left[1 - \left(\frac{B_m}{B_m^{(c)}}\right)^2\right]}, \tag{1}$$

where α is the surface tension or the surface energy per unit area of the critical superconducting bubbles, (from this expression it is possible to obtain the critical size of the quark matter bubble by considering $10^{-3} \leq \alpha 1$ as the range of surface tension) which is greater than zero for a type-I superconductor-normal interface, $B_m^{(c)}$ is the critical magnetic field. In presence of a magnetic field $B_m < B_m^{(c)}$, the normal to superconducting transition is first order in nature. As the superconducting phase grows continuously, the magnetic field lines will be pushed out into the normal quark matter crust. This is the usual *Meissner effect*, observed in type-I superconductor. We compare this phenomenon of magnetic

flux expulsion from a growing superconducting quark matter core with the diffusion of impurities from the frozen phase of molten metal or the transport of baryon numbers from hot quark matter soup during quark-hadron phase transition in the early universe. The formation of superconducting zone is compared with the solidification of molten metal or with the transition to hadronic phase with almost zero baryon number. It is known from the simple thermodynamic calculations that if the free energy of molten phase decreases in presence of impurity atoms, then during solidification they prefer to reside in the molten phase otherwise they go to the solid phase. In this particular case the magnetic field lines play the role of impurity atoms and because of Meissner effect, they prefer to remain in normal quark matter phase, the normal quark matter phase plays the role of molten metal or the hot quark soup, whereas the superconducting phase can be compared with the frozen solid phase or the hadronic phase. (This idea was applied to baryon number transport during first order quark-hadron phase transition in the early Universe, where baryon number replaces impurity, quark phase replaces molten metal and hadronic matter replaces that of solid metal [26,27]. The baryon number prefers to stay in the quark phase because of Boltzmann suppression factor in the hadronic phase). Since the magnetic flux lines prefer to reside in the normal phase, the well known Meissner effect can therefore be restated as *the solubility of magnetic flux lines in the superconducting phase is zero with a finite penetration depth.*

The dynamical equation for the flux expulsion can be obtained from the simplified model of sharp normal-superconducting interface. The expulsion equation is given by the well known diffusion equation [30]

$$\frac{\partial B_m}{\partial t} = D\nabla^2 B_m \tag{2}$$

where B_m is the magnetic field intensity and D is the diffusion coefficient, given by

$$D = \frac{c^2}{4\pi\sigma_n} \tag{3}$$

where σ_n is the electrical conductivity of the normal quark matter phase with $B_m = 0$ and the symbol c is the velocity of light.. The electrical conductivity of quark matter for $B_m = 0$ is given by [31,32]

$$\sigma_n = 5.8 \times 10^{25} \left(\frac{\alpha_s}{0.1}\right)^{-3/2} T_{10}^{-2} \left(\frac{n}{n_0}\right)$$

whereas the more appropriate form is [33]

$$\sigma_n \sim (\alpha_s T)^{-5/3} \tag{4}$$

and is expressed in sec^{-1}, where α_s is the strong coupling constant and $T_{10} = T/10^{10}$K, the numerical value for this electrical conductivity in the case of quark matter relevant for quark star density is $\sim 10^{26}$ sec^{-1}. We have used this expression to get an order of magnitude estimate of electrical conductivity of quark matter. In actual calculation one has to evaluate σ_n in presence of B_m. In that case, σ_n may not be a scalar quantity (magnetic field destroys the isotropy of the electromagnetic properties of the medium). In particular, for extremely large B_m, the components of electric current vector orthogonal to B_m become almost zero. Which indicates that the quarks can only move along the direction of magnetic field or in other words, across the field resistivity becomes infinity. Here, by normal quark matter we mean that it is non-superconducting in nature. Further, while calculating the electrical conductivity, we have assumed that quark matter is in β-equilibrium. In such condition, since the mass of s-quark is assumed to be 150Mev, the electron density is not exactly zero, but it is a few orders of magnitude less than the s-quark density. Therefore, we can neglect the effect of electrical conductivity from electrons. We have also noticed, that the kinetic energy of these electrons are very high and as a consequence, we can not have superconducting transition of electrons in quark stars.

A solution of eqn.(2) with spherical symmetry can be obtained by Greens' function technique, and is given by (for a general topological structure, no analytical solution is possible)

$$B_m(r,t) = \frac{1}{2r(\pi D t)^{1/2}} \int_0^\infty B_m^{(0)}(r') \left[\exp(-u_-^2) - \exp(-u_+^2)\right] r' dr' \tag{5}$$

where $u_\pm = (r \pm r')/2(Dt)^{1/2}$ and $B_m^{(0)}(r)$ is the magnetic field distribution within the star at $t = 0$, which is of course an entirely unknown function of radial coordinate r. To obtain an estimate of magnetic field diffusion time scale (τ_D), we assume $B_m^{(0)}(r) = B_m^{(0)} =$ constant (in reality, this may not be true inside the star, e.g., in some work we took a parametric form, given by

$$B_m(n_B) = B_m^{\rm surf.} + B_0 \left[1 - \exp\left(-\beta \left(\frac{n_B}{n_0}\right)^\gamma\right)\right]$$

with $\beta = 0.01$, $\gamma = 3$, $B_0 = 5 \times 10^{18}$G and $B_m^{\rm Surf.} \approx 10^{14}$. We have used this radial form of distribution to study the mean field properties of dense quark matter in presence of strong quantising magnetic field [34,35]. However, such a parametrisation is very difficult to use in this particular case). Then using the expression for electrical conductivity, given by eqn.(4), we have $\tau_D = 10^5 - 10^6$yrs. Then with the constant $B_m^{(0)}(r)$, we have from eqn.(5)

$$B_m(r,t) = B_m^{(0)} \left[1 - \frac{2}{r}(\pi Dt)^{1/2}\right] \tag{6}$$

where we have used $\sigma_n = 10^24{\rm sec}{-1}$ Hence putting $B_m(r,t) = 0$, the estimated time scale for the expulsion of magnetic lines is $\sim 10^5 - 10^6$yrs. Which can also be obtained from stability analysis of planer normal-superconducting interface. From this simple analysis, we may therefore conclude that, except the time scale for expulsion of magnetic field lines, almost nothing can be inferred, particularly about the growth of superconducting zone and the associated expulsion of magnetic flux lines from this region by solving eqn.(2). The reason behind such uncertainty is our lack of knowledge or definite ideas on the numerical values of the parameters present in eqn.(6). To get some idea of magnetic field expulsion time scale and the structure of the growing superconducting zone, we shall now investigate the morphological instability of normal-superconducting interface of quark matter using the idea of solute diffusion during solidification of alloys. Since the motion of normal-superconducting interface is extremely important in this case and has to be taken into consideration, then instead of eqn.(2) which is valid in the rest frame, an equation expressed in a coordinate system which is moving with an element of the boundary layer is the correct description of such superconducting growth, known as *Directional Growth*. The equation is called *Directional Growth Equation*, and is given by

$$\frac{\partial B_m}{\partial t} - v\frac{\partial B_m}{\partial z} = D\nabla^2 B_m \tag{7}$$

where the motion of the plane interface is assumed to be along z-axis and v is the velocity of the front. This diffusion equation must be supplemented by the boundary conditions at the interface. The first boundary condition is obtained by combining Ampere's and Faraday's laws at the interface, and is given by

$$B_m v \mid_s = -D(\nabla B_m).\hat{n} \mid_s \tag{8}$$

where $\hat{n}$ is the unit vector normal to the interface directed from the normal phase to the superconducting phase. This is nothing but the continuity equation for magnetic flux diffusion. The rate at which excess magnetic field lines are rejected from the interior of the phase is balanced by the rate at which magnetic flux lines diffuses ahead of the two-phase interface. This effect makes the boundary layer between superconducting-normal quark matter phases unstable due to excess magnetic field lines present on the surface of the growing superconducting bubble. Local thermodynamic equilibrium at the interface gives (Gibbs-Thompson criterion)

$$B_m \mid_s \approx B_m^{(c)} \left(1 - \frac{4\pi\alpha}{R B_m^{(c)^2}}\right) = B_m^{(c)} \left(1 - \delta C\right) \tag{9}$$

where δ is called capillary length with α the surface tension, C is the curvature $= 1/R$ (for a spherical surface), and $B_m^{(c)}$ is the thermodynamic critical field.

To investigate the stability of superconducting-normal interface, we shall follow the original work by Mullins and Sekerka [28,29], and consider a steady state growth of superconducting core, then the

time derivative in eqn.(7) will not appear. Introducing $r_\perp = (x^2+y^2)^{1/2}$ as the transverse coordinate at the interface, then we have after rearranging eqn.(7)

$$\left[\frac{\partial^2}{\partial r_\perp^2} + \frac{1}{r_\perp}\frac{\partial}{\partial r_\perp} + \frac{\partial^2}{\partial z^2} + \frac{v}{D}\frac{\partial}{\partial z}\right] B_m = 0 \tag{10}$$

The approximation that the solidification is occurring under steady state condition used in the freezing of molten material will be followed in the present case of normal to superconducting phase transition. Now if it is assumed that these two phenomena taking place in two completely separate physical world are almost identical natural processes, then the concentration of magnetic flux lines and normal-superconducting interface morphology will be independent of time. The main disadvantage of this assumption is that there will be no topological evolution of the interface shape. As a consequence of this constraint the solution to the basic diffusion problem is indeterminate and a whole range of morphologies is permissible from the mathematical point of view. In order to distinguish the solution which is the most likely to correspond to reality, it is necessary to find some additional criteria. The examination of the stability of a slightly perturbed growth form is probably the most reasonable manner in which this situation may be treated. In the following we shall investigate the morphological instability of normal-superconducting interface from eqn.(10). Assuming a solution of this equation expressed as the product of separate functions of $r_\perp$ and z and setting the separation constant equal to zero and using the boundary condition given by eqn.(9), we have for an unperturbed boundary layer moving along z-axis

$$B_m = B_m^{(s)} \exp(-zv/D) = B_m^{(s)} \exp(-2z/l) \tag{11}$$

where $l = 2D/v$ is the layer thickness, which is very small for small D. Mathematically, the thickness of this layer is infinity. For practical purpose an effective value l can be taken. The order of magnitude estimates or limiting values for the three quantities D, v and l can be obtained from the stability condition of planer interface, which will be discussed latter.

Due to excess magnetic flux lines at the interface, the form of the planer normal-superconducting interface described by the equation $z = 0$ is assumed to be changed by a small perturbation represented by the simple sine function

$$z = \epsilon \sin(\mathbf{k}.\mathbf{r}_\perp) \tag{12}$$

where ϵ is very small amplitude and $\mathbf{k}$ is the wave vector of the perturbation. Then the perturbed solution of the magnetic field distribution near the interface can be written as

$$B_m = B_m^{(s)} \exp(-vz/D) + A\epsilon \sin(\mathbf{k}.\mathbf{r}_\perp) \exp(-bz) \tag{13}$$

where A and b are two unknown constants. Since the solution should satisfy the diffusion equation (eqn.(10)), we have

$$b = \frac{v}{2D} + \left[\left(\frac{v}{2D}\right)^2 + k^2\right]^{1/2} \tag{14}$$

To evaluate A, we utilise the assumption that ϵ and $\epsilon\sin(\mathbf{k}.\mathbf{r}_\perp)$ are small enough so that we can keep only the linear terms in the expansion of exponentials present in eqn.(13). Then at the interface, we have after some straight forward algebraic manipulation

$$A = \frac{v}{D} B_m^{(s)} \tag{15}$$

The expression describing the magnetic field distribution ahead of the slightly perturbed interface then reduces to

$$B_m = B_m^{(s)} \left[\exp(-vz/D) + \frac{v}{D}\epsilon \sin(\mathbf{k}.\mathbf{r}_\perp) \exp(-bz)\right] \tag{16}$$

Now from the other boundary condition (eqn.(9)) we have

$$B_m^{(s)} = B_m^{(c)} - \frac{4\pi\alpha B_m^{(c)}}{B_m^{(s)2}} C \tag{17}$$

where $C = z^{''}/(1+z^{'2})^{3/2}$ is the curvature at $z = \epsilon\sin(\mathbf{k}.\mathbf{r}_\perp)$ and prime indicates derivative with respect to $r_\perp$.

Neglecting z'^2, which is small for small perturbation, we have

$$B_m^{(s)} = B_m^{(c)} + \Gamma k^2 S \tag{18}$$

where $\Gamma = 4\pi\alpha B_m^{(c)}/B_m^{(s)2}$ and we have replaced $\epsilon \sin(\mathbf{k}.\mathbf{r}_\perp)$ by S. Since the amplitude of perturbation ϵ is extremely small, the quantity S is also negligibly small.

Now the eqn.(18) is also given by

$$B_m^{(s)} = B_m^{(c)} + GS \tag{19}$$

where

$$G = \frac{dB_m}{dz} \mid_{z=S} = -\frac{v}{D}\left(1 - \frac{vS}{D}\right) B_m^{(s)} - bAS(1 - bS) \tag{20}$$

Combining these two equations, we have

$$k^2\Gamma + \frac{v}{D}\left(1 - \frac{vS}{D}\right) B_m^{(s)} - \frac{bv}{D} B_m^{(s)} S(1 - bS) = 0 \tag{21}$$

This expression determines the form (values of k) which the perturbed interface must assume in order to satisfy all of the conditions of the problem. To analyse the behaviour of the roots, we replace right hand side of eqn.(21) by some parameter $-P$. (We have taken $-P$ in order to draw a close analogy with the method given in refs.[26,27]). Then rearranging eqn.(21), we have

$$-k^2\Gamma - \frac{v}{D}\left(1 - \frac{vS}{D}\right) B_m^{(s)} + \frac{bvB_m^{(s)}S}{D}(1 - bS) = P \tag{22}$$

(in refs.[26,27] the parameter P is related to the time derivative of ϵ, the amplitude of small perturbation). If the parameter P is positive for any value of k, the distortion of the interface will increase, whereas, if it is negative for all values of k, the perturbation will disappear and the interface will be stable. In order to derive a stability criterion, it only needs to know whether eqn.(20) has roots for positive values of k. If it has no roots, then the interface is stable because the $P - k$ curve never rises above the positive k-axis and P is therefore negative for all wavelengths. We have used Decarte's theorem to check how many positive roots are there. It is more convenient to express k in terms of b and then replacing b by $\omega + v/D$, we have from eqn.(22)

$$\begin{aligned} -\omega^2\left(\Gamma + \frac{vB_m^{(s)}S^2}{D}\right) - \omega\left(\Gamma + \frac{2vB_m^{(s)}S^2}{D} - B_m^{(s)}S\right)\frac{v}{D} \\ - \frac{v}{D}B_m^{(s)}\left(1 - \frac{v}{D}S\right)^2 = P \end{aligned} \tag{23}$$

This is a quadratic equation for ω. The first and the third terms are always negative. The second term will also be negative if

$$\Gamma + \frac{2vB_m^{(s)}S^2}{D} - B_m^{(s)}S > 0 \tag{24}$$

Then it follows from Decart's rule that if the condition (25) is satisfied, there can not be any positive root. Which implies that the small perturbation of the interface will disappear. Since the amplitude of perturbation is assumed to be extremely small, the quantity $S = \epsilon \sin(\mathbf{k}.\mathbf{r}_\perp)$ is also negligibly small. Under such circumstances the middle term of eqn.(23) is much smaller than rest of the terms. The Decart's rule given by the condition (24) can be re-written as

$$\Gamma > B_m^{(s)}S \tag{25}$$

Which after some simplification gives the stability criterion for the plane unperturbed interface, given by

$$\alpha > \frac{B_m^{(s)3}S}{4\pi B_m^{(c)}} \tag{26}$$

From the stability criterion, it follows that the normal-superconducting interface energy/area of quark matter has a lower bound, which depends on the interface magnetic field strength, critical field

strength and also on the perturbation term S. An order of magnitude of this lower limit can be obtained by assuming $B_m^{(s)} = 10^{-3} B_m^{(c)}$. (Since the critical field $B_m^{(c)} \sim 10^{16}$G, and the neutron star magnetic field strength $B_m \sim 10^{13}$G, we may use this equality). Then the lower limit is given by

$$\alpha_L \approx 10^{-9} \ \ \mathrm{MeV/fm}^2 \ \left(\frac{S}{\mathrm{fm}}\right) \tag{27}$$

On the other hand for $B_m^{(s)} = 0.1 B_m^{(c)}$, we have

$$\alpha_L \approx 10^{-3} \ \ \mathrm{MeV/fm}^2 \ \left(\frac{S}{\mathrm{fm}}\right) \tag{28}$$

The approximate general expression for the lower limit is given by

$$\alpha_L \approx h^3 \ \mathrm{MeV/fm}^2 \ \left(\frac{S}{\mathrm{fm}}\right) \tag{29}$$

where $h = B_m^{(s)}/B_m^{(c)}$. Therefore the maximum value of this lower limit is

$$\alpha_L^{\mathrm{max.}} \approx 1 \ \mathrm{MeV/fm}^2 \ \left(\frac{S}{\mathrm{fm}}\right) \tag{30}$$

when the two phase are in thermodynamic equilibrium. Of course for such a strong magnetic field, as we have seen [6,7] that there can not be a first order quark-hadron phase transition.

On the other hand if we do not have control on the interface energy, which can in principle be obtained from Landau-Ginzberg model, we can re-write the stability criteria in terms of interface concentration of magnetic field strength $B_m^{(s)}$, and is given by

$$B_m^{(s)} < \left[\frac{4\pi\alpha B_m^{(c)}}{S\left(1 - \frac{2v}{D} S\right)}\right]^{1/3} \tag{31}$$

This is more realistic than the condition imposed on the surface tension α. Now for a type-I superconductor, the surface tension $\alpha > 0$, which implies $1 - 2vS/D > 0$. Therefore, we have $2vS/D < 1$, and for the typical value $\sigma_n \sim 10^{26}$ sec^{-1} for the electrical conductivity of normal quark matter, the profile velocity $v < D/2S \sim 10^{-6}/S$cm/sec $\sim$ 1cm/sec for $S \sim 10^{-6}$cm. Therefore the interface velocity $<$ 1cm/sec for such typical values of σ_n and S to make the planer interface stable under small perturbation. Now the thickness of the layer at the interface is $l = 2D/v > 10^{-6}$cm for such values of D (or σ_n) and v. Here S is always greater than 0, otherwise, the magnetic field strength at the normal-superconductor interface becomes unphysical. As before, if the second term of eqn.(23) is negligibly small compared to other two terms, we have

$$B_m^{(s)} < \left[\frac{4\pi\alpha B_m^{(c)}}{S}\right]^{1/3} \tag{32}$$

3 Conclusion

Therefore we may conclude that if a superconducting transition occurs in a quark star, the magnetic properties of such bulk object are entirely different from that of a small laboratory superconducting sample. Expulsion of magnetic flux lines from the superconducting zone is not at all instantaneous. The typical time scale is $10^5 - 10^6$ yrs. We have noticed that this time scale is very close to the magnetic field decay time scale in a neutron star. Due to the presence of excess magnetic flux lines at the interface, which is true if the diffusion rate of magnetic lines of forces in the normal phase is less than the rate of growth of the superconducting zone, the topological structure of normal-superconducting boundary layer may change significantly. Of course, it depends on the magnitude of surface tension α. It may take dendritic shape instead of planer structure. The stability of planer interface also depends on the strength of interface magnetic field at the boundary layer. Since the expulsion time scale is very high,

we expect that there will be no instability at the interface between normal and superconducting quark matter phase. The superconducting phase will grow steadily. How to get evidence from observational data for such an unusual shape of normal-superconducting quark matter interface is a matter of further study.

Finally, we know that in the laboratory, since the size of the superconducting sample is about a few cm, the expulsion is instantaneous, if there is a type one transition to superconducting phase. On the other hand, the problem we are dealing with is having a linear dimension of a few Km, therefore, it is quite obvious, that the expulsion can not be instantaneous nature and the model of impurity diffusion in molten alloy, we are employing here is one of the techniques to study such transition in bulk phase. We believe, that the model is also applicable to any bulk system of charged fermions, where a type one super-conducting transition is occurring.

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